Effect of Viscous Dissipation and Soret on MHD Flow with Thermal Radiation

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Abstract

This study considers heat and mass transfer of viscous, incompressible and electrically conducting fluid with viscous dissipation, Soret and thermal radiation. The coupled partial differential equations for momentum, energy and concentration were transformed into ordinary differential equations by assuming purely oscillatory flow. The resulting equations were then solved analytically. Using realistic values for the parameters entering the problem, the solutions were displayed graphically and discussed. The results show that velocity, temperature and concentration were appreciably influenced by the magnetic field, Soret, permeability, thermal radiation, Eckert number and Reynolds number. It is noted that Reynolds number and permeability parameter enhances the velocity while magnetic field and thermal radiation decreases the velocity. The Eckert number increases the temperature while the Prandtl number decreases it. Magnetic field, Soret and Schmidt number enhance the concentration. The Eckert number increases both heat and mass transfer rates.

Keywords: Magnetohydrodynamics, thermal radiation, Soret, viscous dissipation, porous medium.

1. Introduction

The flow of fluid through porous medium past vertical plate especially in heat transfer situations is common in nature and has many applications in science and engineering. The heating of rooms and buildings, general cooling system design, heat exchangers are some examples. Heat transfer by thermal radiation is important when concerned with higher operating temperatures, more so, the interplay of magnetic field and thermal radiation in porous medium have useful applications in astrophysics, geophysical fluid dynamics, radio propagation through the ionosphere, MHD pumps, nuclear power plants, gas turbines and space technology.

Kim (2000) studied unsteady MHD convective heat transfer over a vertical porous plate with variable suction. Pal (2013) considered Hall current and MHD effects on an unsteady stretching permeable surface with thermal radiation. Abd El- Naby *et al* (2003) considered radiation effects on MHD free convection flow with variable surface temperature. Their study considered a finite difference solution to the problem. Hossain and Tekha (1996) studied the interaction of radiation with mixed convection flows past a vertical plate. Jha and Ajibade (2011) studied heat generation and absorption on natural convection between two infinite plates subjected to periodic heating. At high Prandtl number, fluid viscosity due to temperature may affect flow characteristics and functioning of industrial machinery where lubrication is essential. The study of heat and mass transfer effect on MHD flow of a visco-elastic fluid with oscillatory suction and heat source was carried out by Mishra *et al* (2013). Bister and Emmanuel (1998) showed that viscous dissipation is a significant heat source in hurricanes and that it increases its

efficiency. The effects of viscous dissipation and radiation on the thermal boundary layer over a non-linearly stretching sheet was studied by Cortell (2008). Berletta and Rossi di Schio (2001) studied the influence of viscous dissipation in a vertical tube with uniform heat flux. Sonth *et al* (2002) considered heat and mass transfer in a visco-elastic flow with heat source/sink and viscous dissipation. Seth and Ansari (2010) considered the presence of thermal diffusion and heat convection flow past an impulsively moving vertical plate with ramped wall temperature. Raptis and Perdikis (2006) studied chemical reaction and magnetic effects in their work on viscous flow over a nonlinearly stretching. Patil *et al* (2013) studied chemical reaction effects on unsteady mixed convection boundary layer flow due to a nonlinearly stretching velocity. The use of perturbation analysis was employed by Pal and Talukdar (2010) to study MHD convective heat and mass transfer in a boundary layer slip flow with thermal radiation and chemical effects. Other important researches involving chemical reaction include Kandaswamy *et al* (2005) and Mahdy (2010).

Thermophoresis or Soret is a condition observed in mixtures of mobile particles where different particles exhibit different responses to the forces of temperature gradient. This phenomenon has applications in electrostatic precipitation, separation of polymer particles in fluid flow fractionation. The studies of Ahmad (2010), Anghel and Tekhar (2000), Amos *et al* (2018) are relevant in this respect.

The motion of conducting fluid across a magnetic field and the interaction arising therefrom generates mechanical forces which modify the flow of the fluid. Such interactions, most times, occur at high temperatures, hence the essence of magnetic field and thermal radiation. We also note that many transport processes exist in nature and in industrial applications in which heat and mass transfer occur as a result of combined buoyancy effects of diffusion and chemical reaction. Consequently, in this study we explore the effect of viscous dissipation on MHD convective flow with thermal radiation.

2. Mathematical Formulation

We consider the flow of an unsteady electrically conducting, viscous and incompressible fluid through two vertical porous plates which are infinite in length and at distance 'd' apart. The Cartesian coordinate is introduced with the origin at the stationary plate with constant injection velocityV. The x-axis is along the plate and the y-axis perpendicular to it. The other plate moves with uniform velocity U and subjected to constant suction velocity V. A uniform magnetic field of strength B_0 is applied normal to the plates. The Reynolds number is assumed small such that the induced magnetic field is neglected. The physical properties of the fluid are functions of y' and t'. Taking into account viscous dissipation heating within the fluid, the governing equations for the flow are;

$$\frac{\partial v'}{\partial v'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + \nu' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial P'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial {y'}^2} - \sigma B_0^2 u' - \nu \frac{u'}{\kappa}$$
(2)

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q'}{\partial y'} + \frac{\mu}{\rho c_p} \left(\frac{\partial u'}{\partial y'}\right)^2$$
(3)

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial {y'}^2} + D_1 \frac{\partial^2 T'}{\partial {y'}^2}$$
(4)

where P' is the pressure, ρ is the density of the fluid, ν is kinematic viscosity, t is time, K is the permeability of the porous medium, c_p the is specific heat capacity at constant pressure, D is mass diffusivity, q' is the radiation heat flux. The boundary conditions are International Journal of Applied Science and Mathematical Theory ISSN 2489-009X Vol. 5 No. 1 2019 www.iiardpub.org

$$u' = 0, \quad T' = T'_0, \quad C' = C'_0 \tag{5}$$

$$u' = U, \quad T' = T'_w, \quad C' = C'_w \tag{6}$$

We assume that the fluid is optically thin with relatively low density such that the radiative heat flux is expressed as

$$\frac{\partial q'}{\partial y'} = 4\alpha^2 T' \tag{7}$$

Where α is the mean radiation absorption coefficient. By the assumption of constant injection and suction velocity V, equation (1) integrates (8)

$$v' = I$$

The non dimensional variables for the problem are'

 $x = \frac{x'}{d}, \quad y = \frac{y'}{d}, \quad u = \frac{u'}{U}, \quad t = t'\frac{V}{d}, \quad T = \frac{T' - T'_0}{T'_w - T'_0}, \quad C = \frac{C' - C'_0}{C'_w - C'_0}, \quad P = \frac{P'}{\rho UV}, \quad K = \frac{K'}{d^2}$ (9)In view of equation (8) and the substitution of equation (9) into equations (2) - (6) we have:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{Re} u - \frac{1}{KRe} u$$
(10)

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y} = \frac{1}{\Pr Re} \frac{\partial^2 T}{\partial y^2} - \frac{N^2}{Re} T + \frac{Ec}{Re} \left(\frac{\partial u}{\partial y}\right)^2$$
(11)

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial y} = \frac{1}{Re \, Sc} \frac{\partial^2 C}{\partial y^2} + \frac{So}{Re} \frac{\partial^2 T}{\partial y^2} \qquad (12)$$

The boundary conditions becomes:

$$u = 0, T = 0, C = 0$$
 at $y = 0$ (13)

$$u = 1, T = 1, C = 1$$
 at $y = 1$ (14)

where,
$$Sc = \frac{v}{D}$$
, $Pr = \frac{\mu c_p}{\kappa}$, $N = 2\alpha \frac{d}{\sqrt{\kappa}}$, $Re = \frac{Vd}{v}$, $So = \frac{D_1(T'_w - T'_0)}{v(C'_w - C'_0)}$, $Ec = \frac{U^2}{c_p T_0}$ (15)

3. Solution of the problem

To solve equations (10) - (14) for purely oscillatory flow, we assume the solution in the form $u(y,t) = u_0(y)e^{i\omega t}, \ T(y,t) = \theta_0(y)e^{i\omega t}, \ \phi(y,t) = \phi_0(y)e^{i\omega t}, \ -\frac{\partial P}{\partial x} = Pe^{i\omega t}$ (16)where ω is the frequency of oscillation and *P* is constant pressure. Substitution of equation (16) into equations (10) - (14), we obtain: $u_{0}'' - Re u_{0}' - (M^{2} + \frac{1}{2} + i\omega Re)u_{0} = -PRe$ (17)

$$b_0 - ReFI b_0 - (N + i\omega ReFI) b_0 - -LCFI u_0$$

$$b_0'' - ReSc b_0' - Sc Sc A_0''$$

$$(10)$$

$$\phi_0^{\prime\prime} - Re\,Sc\,\phi_0^{\prime} - \,i\omega\,Re\,Sc\,\phi_0 = -Sc\,So\,\theta_0^{\prime\prime} \tag{19}$$

Subject to:
$$y_{1} = 0$$
, $\theta_{1} = 0$, $\phi_{2} = 0$, at $y_{1} = 0$

$$u_0 = 0, \ \theta_0 = 0, \ \phi_0 = 0$$
 at $y = 0$ (20)
 $u_0 = 1, \ \theta_0 = 1, \ \phi_0 = 1$ at $y = 1$ (21)

The solutions to equations (17) - (19) subject to (20) and (21) and taking note of equation (16) are:

$$u = \left\{ \left(-D_2 - \frac{PRe}{D_1} \right) e^{\beta_1 y} + D_2 e^{\beta_2 y} + \frac{PRe}{D_1} \right\} e^{i\omega t}$$
(22)

$$T = \left\{ E_8 e^{\beta_3 y} + E_9 e^{\beta_4 y} + E_5 e^{2\beta_1 y} + E_6 e^{(\beta_1 + \beta_2) y} + E_7 e^{2\beta_2 y} \right\} e^{i\omega t}$$
(23)

$$C = \left\{ E_{15}e^{\beta_5 y} + E_{16}e^{\beta_6 y} + E_{10}e^{\beta_3 y} + E_{11}e^{\beta_4 y} + E_{12}e^{2\beta_1 y} + E_{13}e^{(\beta_1 + \beta_2)y} + E_{14}e^{2\beta_2 y} \right\} e^{i\omega t}$$
(24)

where

$$\begin{split} D_1 &= M^2 + \frac{1}{\kappa} + i\omega \; Re, \; \beta_1 = \frac{Re - \sqrt{Re^2 + 4D_1}}{2}, \\ \beta_2 &= \frac{Re + \sqrt{Re^2 + 4D_1}}{2}, \\ D_2 &= \frac{D_1 + P \; Re(e^{\beta_1 - 1})}{D_1(e^{\beta_2} - e^{\beta_1})} \;, \\ D_3 &= 4(N^2 + i\omega \; Re \text{Pr}), \; \beta_3 = \frac{RePr + \sqrt{(RePr)^2 + D_3}}{2}, \\ \beta_4 &= \frac{RePr - \sqrt{(RePr)^2 + D_3}}{2}, \end{split}$$

$$\begin{split} E_5 &= \frac{-Ec \Pr \beta_1^2 \left(-D_2 - \frac{PRe}{D_1}\right)^2}{4\beta_1^2 - 2Re \Pr \beta_1 - D_3}, \ E_6 &= \frac{-2Ec \Pr D_2 \beta_2 \beta_1}{(\beta_1 + \beta_2)^2 - Re \Pr (\beta_1 + \beta_2) - D_3}, \ E_7 &= \frac{-2Ec \Pr D_2^2 \beta_3^2}{\beta_2^2 - 2Re \Pr \beta_2 - D_3}, \\ \beta_5 &= \frac{ReSc + \sqrt{(ReSc)^2 + 4(i\omega Sc)}}{2}, \ \beta_6 &= \frac{ReSc - \sqrt{(ReSc)^2 + 4(i\omega Sc)}}{2}, \ E_8 &= -E_9 - E_5 - E_6 - E_7, \\ E_9 &= 1 + (E_5 + E_6 + E_7)e^{\beta_3} - E_5e^{2\beta_1} - E_6e^{(\beta_1 + \beta_2)} - E_7e^{2\beta_2}, \ E_{10} &= \frac{E_8\beta_3^2Sc So}{\beta_3^2 - Re Sc \beta_3 - i\omega Re Sc}, \\ F_{11} &= \frac{E_9\beta_4^2Sc So}{\beta_4^2 - Re Sc \beta_4 - i\omega Re Sc}, \ E_{12} &= \frac{4E_5\beta_1^2Sc So}{4\beta_1^2 - Re Sc \beta_1 - i\omega Re Sc}, \\ E_{13} &= \frac{E_6(\beta_1 + \beta_2)^2Sc So}{(\beta_1 + \beta_2)^2 - Re Sc (\beta_1 + \beta_2) - i\omega Re Sc}, \\ E_{14} &= \frac{4E_7\beta_2^2Sc So}{4\beta_1^2 - Re Sc \beta_1 - i\omega Re Sc}, \ E_{15} &= -E_{16} - E_{10} - E_{11} - E_{12} - E_{13} - E_{14}, \\ E_{16} &= \frac{1 + (E_{10} + E_{11} + E_{12} + E_{13} + E_{14})e^{\beta_5} - E_{10}e^{\beta_3} - E_{12}e^{\beta_4} - E_{12}e^{2\beta_1} - E_{13}e^{(\beta_1 + \beta_2)} - E_{14}e^{2\beta_2}} \end{split}$$

The skin friction, C_f , the Nusselt number, Nu and Sherwood number, Sh are given respectively as

$$C_f = \left(\frac{\partial u_o}{\partial y}\right)_{y=0} = \beta_1 \left(-D_2 - \frac{PRe}{D_1}\right) + D_2 \beta_2$$
(25)

$$Nu = \left(\frac{\partial \theta_0}{\partial y}\right)_{y=0} = E_8 \beta_3 + E_9 \beta_4 + 2E_5 \beta_1 + E_6 (\beta_1 + \beta_2) + 2E_7 \beta_2$$
(26)

$$Sh = \left(\frac{\partial\phi_0}{\partial y}\right)_{y=0} = E_{15}\beta_5 + E_{16}\beta_6 + E_{10}\beta_3 + E_{11}\beta_4 + E_{12}2\beta_1 + E_{13}(\beta_1 + \beta_2) + E_{14}2\beta_2$$
(27)

4. Discussion of Results

Figure 1 shows the effect of magnetic field on velocity. The profiles indicate that increase in magnetic field decreases the velocity. Physically, the presence of magnetic field introduces the Lorentz force which acts against the flow, hence the decrease in velocity. The influence of the Reynolds number on the velocity is shown in Figure 2. It is observed that increase in the Reynolds number enhances the velocity. Basically for large values of the Reynolds number, the inertial forces are predominant. The effect of the permeability of the medium on velocity is depicted in Figure 3. It shows that increase in the permeability increases the fluid flow. Physically this is possible as permeability is a property of the porous medium with its increase indicating the ability of the formation to transmit more fluid. Figure 4 depicts the effect of magnetic field on the temperature. The profile indicates that increase in the magnetic field applied heats up the fluid thereby increasing the temperature.

The effect of the Eckert number on the temperature distribution is shown in Figure 5. Increase in Eckert number show increase in the temperature of the flow field. This is because additional heat is added to the system by increasing the Eckert number. Figure 6 shows the influence of radiation on the temperature. The indication from the profile is that increase in thermal radiation decreases the temperature. Physically, large values of thermal radiation indicates an increase in dominance of conduction in the system hence decrease in the buoyancy force. The influence of the permeability of the medium on temperature is shown in Figure 7. It is observed that increase in permeability decreases the temperature. Increase in the permeability pushes more amount of fluid into the flow field hence the flow field suffers a decrease in the Prandtl number decreases the temperature. The increase in the Prandtl decreases thermal conductivity while increasing the viscosity, this has the effect of decreasing the thermal boundary layer

thickness hence the decrease in temperature. The profile indicates that the effect of increase in Prandtl number is more pronounced far from the plate.

Figure 9 depicts concentration profile for various values of magnetic field. It shows that increase in magnetic field leads to higher concentration. This confirms that a weak magnetoconvective force leads to low concentration. The effect of Soret on the concentration of the fluid is shown in Figure 10. It is noted that increase in Soret enhances the concentration. This is due to increase in thermal diffusion. The profile indicate significant effect on the concentration even at minimal increase in the Soret. The influence of the Schmidt number on the concentration is indicated in Figure 11. The profile indicate that increase in the Schmidt number increases the concentration. The effect of thermal radiation on concentration is depicted in Figure 12. It shows that increase in thermal radiation increases the concentration. Figure 13 shows the effect of magnetic field on the skin friction. It is noted that increase in the magnetic field decreases the skin friction. Expectedly this is due to the drag at the boundary layer. Even with increase in the Reynolds number the effect of the magnetic field remains predominant and unchanged. It is shown in Figure 14 that increase in the Eckert number increases the heat transfer rate, but this effect is decreased down the line as the Reynolds number is increased. Figure 15 shows the effect of the Eckert number on mass transfer. The Eckert number increases mass transfer. The Eckert number exerts a domineering influence over increase in the Reynolds number.





Figure 5: Influence of Eckert number on the temperature for Pr = 0.71, N = 0.5, t = 0, M = 1, Re = 1, K = 0.2



Figure 6: Influence of thermal radiation on the temperature for Pr = 0.71, Ec =0.2, t = 0, M = 1, Re = 1, K = 0.2



Figure 7: Influence of permeability on the temperature for Pr = 0.71, N = 0.5, t = 0, M = 1, Re = 1, N = 0.5



Figure 8: Influence ^y of Prandtl number on the temperature for K = 0.2, N = 0.5, t = 0, M = 1, Re = 1, N = 0.5



Figure 9: Influence of magnetic field on the concentration for K = 0.2, N = 0.5, t = 0, Sc = 0.2, Re = 1, N = 0.5, So = 0.3, Ec = 0.2



Figure 10: Influence of Soret on the concentration for K = 0.2, N = 0.5, t = 0, M = 1, Re = 1, N = 0.5, Sc = 0.2, Ec = 0.2



Figure 11: Influence of Schmidt number on the concentration for = 0.2, N = 0.5, t = 0, M = 1, Re = 1, N = 0.5, So = 0.3, Ec = 0.2



Figure 12: Influence of thermal radiation on the concentration for = 0.2, Sc = 0.2, t = 0, M = 1, Re = 1, N = 0.5, So = 0.3, Ec = 0.2



Ec = 0.1, 0.2, 0.3, 0.5

Figure 14: Influence of Eckert number and Reynolds numberon heat transfer for K = 0.2, N = 0.5, t = 0, M = 1, N = 0.5



Figure 15: Influence of Eckert number and Reynolds number on the concentration for K = 0.2, N = 0.5, t = 0, M = 1, N = 0.5, So = 0.3,

Figure 13: Influence of magnetic field and Reynolds number on the skin friction for K = 0.2, $\omega = 1, t = 0$

5. Conclusion

The problem of heat and mass transfer influenced by magnetic field, thermal radiation and viscous dissipation has been formulated and solved. Employing analytical solution to the problem and adopting parameter values, we have displayed the results graphically and presented the interpretations. Consequently the following conclusion are drawn:

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(i) Magnetic field and thermal radiation retard the velocity of the fluid while the permeability enhance the velocity

(ii) The Eckert number increases the temperature while the Prandtl number decreases it.

(iii) Higher magnetic field leads to higher concentration. Increase in Soret, Schmidt number and thermal radiation increases the concentration.

(iv) Magnetic field decreases the skin friction while the Eckert number increases both heat and mass transfer rates.

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